Column Buckling: Johnson and Euler Formulas

Critical load of a pin-ended or other column by the Johnson parabola for intermediate columns and Euler for long columns, with the slenderness ratio and transition.

Method: Budynas and Nisbett; Shigley's Mechanical Engineering Design; ...

Input parameters

Unit system: SI

Parameter	Value	Unit
Effective length (K L)	1.2000	m
Cross-sectional area	1,257.0	mm²
Least radius of gyration	10.000	mm
Young's modulus	207.00	GPa
Yield strength	250.00	MPa

Calculated results

Parameter	Value	Unit
Slenderness ratio	120.00	
Transition slenderness √(2π² E/S_y)	127.84	
Critical load	175.81	kN
Critical stress	139.87	MPa

Notes for this case

- Intermediate column: Johnson's parabola governs.

Formula and method

$$\left(\frac{L_e}{k}\right)_1 = \sqrt{\frac{2\pi^2 E}{S_y}}, \quad \frac{P_{cr}}{A} = S_y - \left(\frac{S_y L_e}{2\pi k}\right)^2 \frac{1}{E} \ \text{(Johnson)}, \quad \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/k)^2} \ \text{(Euler)}$$

Variables

L_e	Effective length (K L)	m
A	Cross-sectional area	m²
k	Least radius of gyration	m
E	Young's modulus	Pa
S_y	Yield strength	Pa
L_e/k	Slenderness ratio	-
$(L_e/k)_1$	Transition slenderness √(2π² E/S_y)	-
P_{cr}	Critical load	N
P_{cr}/A	Critical stress	Pa

Method and standards

Parabola tangent to Euler's curve at half the yield strength.

Assumptions and limits

- Straight, centrally loaded column.
- P_cr is the failure (critical) load with no factor of safety: divide by your design factor. The Euler-only calculator gives the same long-column result.

References

- Budynas and Nisbett

- Shigley's Mechanical Engineering Design
- 10th ed. (McGraw-Hill, 2015)
- ch. 4 (Euler and J.B. Johnson column formulas)

Report details

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Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/mechanical/strength-of-materials/johnson-column

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.