Curved Beam Bending Stress (Winkler–Bach), Rectangular Section

Neutral axis shift and bending stresses at the inner and outer fibres of a curved beam of rectangular section, as in hooks and C-frames, by the Winkler–Bach theory.

Method: Shigley's Mechanical Engineering Design (Budynas and Nisbett); ch. 3 (curved beams in bending, rectangular section r_n = h / ln(r_o/r_i))

Input parameters

Unit system: SI

Parameter	Value	Unit
Inner radius	50.000	mm
Section depth (radial)	40.000	mm
Section width	20.000	mm
Bending moment (positive straightens the beam)	-1,500.0	N·m

Calculated results

Parameter	Value	Unit
Radius of the centroid	70.000	mm
Radius of the neutral axis h/ln(r_o/r_i)	68.052	mm
Neutral axis shift r_c − r_n	1.9481	mm
Stress at the inner fibre	-347.49	MPa
Stress at the outer fibre	234.72	MPa
Straight-beam stress 6M/(b h²) for comparison	281.25	MPa

Notes for this case

- Sign: a positive moment straightens the beam (tension at the inner fibre). In a crane hook the direct tension P/A is added.

Formula and method

$$r_n = \frac{h}{\ln(r_o/r_i)}, \quad e = r_c - r_n, \quad \sigma_i = \frac{M(r_n - r_i)}{A e r_i}, \quad \sigma_o = \frac{M(r_n - r_o)}{A e r_o}$$

Variables

r_i	Inner radius	m
h	Section depth (radial)	m
b	Section width	m
M	Bending moment (positive straightens the beam)	N·m
r_c	Radius of the centroid	m
r_n	Radius of the neutral axis h/ln(r_o/r_i)	m
e	Neutral axis shift r_c − r_n	m
σ_i	Stress at the inner fibre	Pa
σ_o	Stress at the outer fibre	Pa
$\sigma_{straight}$	Straight-beam stress 6M/(b h²) for comparison	Pa

Method and standards

Winkler–Bach curved beam theory.

Assumptions and limits

- Rectangular section, elastic.

- For large radius-to-depth ratios the shift e is found from the series of ln(r_o/r_i), avoiding the loss of digits in r_c − r_n.
- Bending only: in a crane hook add the direct stress P/A to both fibres.

References

• Shigley's Mechanical Engineering Design (Budynas and Nisbett)
• ch. 3 (curved beams in bending, rectangular section r_n = h / ln(r_o/r_i))

Report details

Report reference	FCC-7AEH-PMCT (the same inputs always give the same reference)
Generated	17 November 2026, 19:50 UTC
Calculator	Curved Beam Bending Stress (Winkler–Bach), Rectangular Section
Method last reviewed	12 November 2026
Software release	FireCAD Calc, release of 17 November 2026
Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/mechanical/strength-of-materials/curved-beam-stress

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Reopens the calculator with the inputs of this report.