Deflection of Curved Bars and Rings (Castigliano)

Deflections of a quarter-circle cantilever under a tip load and of a thin ring under diametral load, by Castigliano's theorem with bending energy only.

Method: Castigliano's second theorem for curved members

Input parameters

Unit system: SI

Parameter	Value	Unit
Member	Quarter-circle cantilever, load at the free end along the fixed end's tangent	
Load	0.50000	kN
Mean radius of the bar	250.00	mm
Elastic modulus	200.00	GPa
Second moment of area of the cross-section	0.83300	cm^4

Calculated results

Parameter	Value	Unit
Deflection in the line of the load	3.6830	mm
Deflection perpendicular to the load (ring: increase in diameter)	2.3447	mm
Largest bending moment	125.00	N·m

Notes for this case

• Castigliano's theorem gives a deflection as the derivative of the strain energy U = ∫M²ds/2EI with respect to a load at that point (a dummy load for other directions). For a quarter-circle cantilever loaded at the tip along the tangent to the fixed end, M = PR sin θ, giving δ = πPR³/4EI along the load and PR³/2EI across it. For a thin ring pinched by two opposite loads, symmetry fixes the unknown moment at M_0 = PR(1/2 − 1/π) and the diameter shortens by (π/4 − 2/π)PR³/EI ≈ 0.149PR³/EI while widening by (2/π − 1/2)PR³/EI ≈ 0.137PR³/EI. Shear and axial energy are neglected (R much larger than the section depth).

Formula and method

$$\delta = \frac{\partial U}{\partial P}, \quad U = \int \frac{M^2}{2EI} ds, \quad \delta_{ring} = \left(\frac{\pi}{4} - \frac{2}{\pi}\right) \frac{PR^3}{EI}$$

Variables

case	Member	-
P	Load	N
R	Mean radius of the bar	m
E	Elastic modulus	Pa
I	Second moment of area of the cross-section	m^4
δ_P	Deflection in the line of the load	m
$\delta_\perp$	Deflection perpendicular to the load (ring: increase in diameter)	m
M_{max}	Largest bending moment	N·m

Method and standards

Curved bars.

Assumptions and limits

- Slender bar; linear elastic; bending energy only; small deflections.

References

Castigliano's second theorem for curved members

Report details

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Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/mechanical/strength-of-materials/curved-bar-castigliano-deflection

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Reopens the calculator with the inputs of this report.