Beam on Elastic Foundation: Deflection, Moment and Soil Pressure

Deflection, bending moment and foundation pressure of a long beam on a Winkler elastic foundation under a concentrated load, by Hetényi's closed-form solution.

Method: Winkler elastic foundation; infinite-beam solution for a concentrated load

Input parameters

Unit system: SI

Parameter	Value	Unit
Concentrated load	300.00	kN
Modulus of subgrade reaction	20.000	MN/m³
Beam width in contact with the foundation	0.60000	m
Elastic modulus of the beam	30.000	GPa
Second moment of area of the beam	625,000	cm⁴
Distance from the load for the local values	1.0000	m
Beam length (load near mid-length)	12.000	m

Calculated results

Parameter	Value	Unit
Characteristic length	2.8117	m
Deflection under the load	4.4457	mm
Bending moment under the load	210.88	kN·m
Foundation pressure under the load	88.914	kPa
Deflection at x	4.0049	mm
Bending moment at x	87.065	kN·m
Distance to the first zero of moment (π/4β)	2.2083	m
βL (long beam above π)	4.2679	

Notes for this case

- Winkler foundation (pressure proportional to the local deflection), infinite beam with the load far from the ends.
- βL lies between π and 2π: end effects are noticeable unless the load is at mid-length.

Formula and method

$$\beta = \sqrt[4]{\frac{k_s b}{4EI}}, \quad y_0 = \frac{P\beta}{2k_s b}, \quad M_0 = \frac{P}{4\beta}, \quad p_0 = k_s y_0$$

$$y(x) = y_0 e^{-\beta x}(\cos\beta x + \sin\beta x), \quad M(x) = M_0 e^{-\beta x}(\cos\beta x - \sin\beta x)$$

Variables

P	Concentrated load	N
k_s	Modulus of subgrade reaction	N/m³
b	Beam width in contact with the foundation	m
E	Elastic modulus of the beam	Pa
I	Second moment of area of the beam	m⁴
x	Distance from the load for the local values	m

L	Beam length (load near mid-length)	m
$1/\beta$	Characteristic length	m
y_0	Deflection under the load	m
M_0	Bending moment under the load	N·m
p_0	Foundation pressure under the load	Pa
$y(x)$	Deflection at x	m
$M(x)$	Bending moment at x	N·m
$x_{M=0}$	Distance to the first zero of moment (π/4β)	m
βL	βL (long beam above π)	-

Method and standards

Hetényi's solution of EI y'''' + k_s b y = 0 for an infinite beam.

Assumptions and limits

- Linear elastic Winkler foundation that also acts in tension; infinite beam.

References

Winkler elastic foundation, infinite-beam solution for a concentrated load

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

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