Semi-Infinite Solid: Temperature after a Sudden Surface Change

Temperature at a depth below a surface suddenly held at a new temperature, and the penetration depth, for walls, soil and thick plates (semi-infinite solid).

Method: Incropera; DeWitt; ...

Input parameters

Unit system: SI

Parameter	Value	Unit
Initial uniform temperature	20.000	°C
Surface temperature held from time zero	100.00	°C
Depth below the surface	20.000	mm
Time	600.00	s
Thermal diffusivity k/(ρ c_p): steel about 12 mm²/s, concrete about 0.7, soil 0.3–0.8.	12.000	mm²/s
Conductivity (for the surface heat flux)	45.000	W/(m·K)

Calculated results

Parameter	Value	Unit
Temperature at the depth	89.411	°C
Similarity variable x/(2√(α t))	0.11785	
Penetration depth 4√(α t)	339.41	mm
Surface heat flux at the time	23.937	kW/m²

Notes for this case

- Valid while the far side is not yet warmed: thickness well beyond the penetration depth.

Formula and method

$$\frac{T(x,t)-T_s}{T_i-T_s}=\operatorname{erf}\frac{x}{2\sqrt{\alpha t}},\quad q''_s=\frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}$$

Variables

T_i	Initial uniform temperature	K
T_s	Surface temperature held from time zero	K
x	Depth below the surface	m
t	Time	s
α	Thermal diffusivity	m²/s
k	Conductivity (for the surface heat flux)	W/(m·K)
$T(x,t)$	Temperature at the depth	K
η	Similarity variable x/(2√(α t))	-
δ_p	Penetration depth 4√(α t)	m
q''_s	Surface heat flux at the time	W/m²

Method and standards

Similarity solution of transient conduction.

Assumptions and limits

- Constant properties, constant surface temperature.

References

- Incropera
- DeWitt
- Bergman and Lavine
- Fundamentals of Heat and Mass Transfer
- ch. 5 (semi-infinite solid, constant surface temperature)

Report details

Report reference	FCC-4W7M-R6B3 (the same inputs always give the same reference)
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Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/mechanical/heat-transfer/semi-infinite-solid-heating

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.