X-Ray Residual Stress by the sin²ψ Method

Surface residual stress from X-ray diffraction peak positions at two tilt angles: lattice spacings by Bragg's law, the d versus sin²ψ slope and the stress.

Method: X-ray diffraction sin²ψ method

Input parameters

Unit system: SI

Parameter	Value	Unit
X-ray wavelength	2.2900	Å
Peak position at ψ = 0	156.40	°
Peak position at tilt ψ	156.00	°
Tilt angle	45.000	°
Elastic modulus (diffraction value)	210.00	GPa
Poisson's ratio	0.29000	

Calculated results

Parameter	Value	Unit
Lattice spacing at ψ = 0	1.1697	Å
Lattice spacing at ψ	1.1706	Å
Strain difference (d_ψ − d₀)/d₀	735.87	με
Residual stress (tension positive)	239.59	MPa

Notes for this case

• Bragg: d = λ/(2 sin θ). For a biaxial surface stress, d varies linearly with sin²ψ and σ_φ = [E/(1 + ν)]·(1/d₀)·∂d/∂sin²ψ; with two tilts the slope is (d_ψ − d₀)/sin²ψ. Use diffraction elastic constants for the reflection measured; more tilts and a least-squares line are better practice, and ψ-splitting means shear stresses are present. A lower 2θ at tilt (larger d) means tension.

Formula and method

$$d = \frac{\lambda}{2\sin\theta}, \quad \sigma_\phi = \frac{E}{1+\nu}\frac{d_\psi - d_0}{d_0 \sin^2\psi}$$

Variables

λ	X-ray wavelength	m
$2\theta_0$	Peak position at ψ = 0	rad
$2\theta_\psi$	Peak position at tilt ψ	rad
ψ	Tilt angle	rad
E	Elastic modulus (diffraction value)	Pa
ν	Poisson's ratio	-
d_0	Lattice spacing at ψ = 0	m
d_ψ	Lattice spacing at ψ	m
ε_ψ	Strain difference (d_ψ − d₀)/d₀	-
σ_ϕ	Residual stress (tension positive)	Pa

Method and standards

Residual stress.

Assumptions and limits

- Biaxial surface stress; two-tilt method; homogeneous material.

References

X-ray diffraction sin²ψ method

Report details

Report reference	FCC-PF8R-JBSS (the same inputs always give the same reference)
Generated	23 November 2026, 17:51 UTC
Calculator	X-Ray Residual Stress by the sin²ψ Method
Method last reviewed	17 November 2026
Software release	FireCAD Calc, release of 22 November 2026
Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/materials/properties/xray-residual-stress-sin2psi

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.