Shaft Critical Speed (Simply Supported, with a Disc)

First critical speed of a simply supported solid round shaft from its own mass and a central disc, by the exact uniform-beam result and Dunkerley's combination.

Method: Euler–Bernoulli beam: ω = (π/L)² √(EI/ρA) for a simply supported shaft; Dunkerley (1894)

Input parameters

Unit system: SI

Parameter	Value	Unit
Span between bearings	1.2000	m
Shaft diameter	50.000	mm
Young's modulus	200.00	GPa
Shaft density	7,850.0	kg/m³
Mass of a disc at mid-span	25.000	kg

Calculated results

Parameter	Value	Unit
Critical speed of the shaft alone	4,129.5	rpm
Critical speed of the disc on a massless shaft	2,493.4	rpm
Combined critical speed (Dunkerley)	2,134.5	rpm

Notes for this case

- Dunkerley gives a lower bound. Stiff-bearing assumption: flexible bearings and supports lower the critical speed further. Keep running speed well away from it.

Formula and method

$$\omega_s = \frac{\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}, \quad \omega_d = \sqrt{\frac{48EI}{mL^3}}, \quad \frac{1}{\omega_c^2} = \frac{1}{\omega_s^2} + \frac{1}{\omega_d^2}, \quad N = \frac{\omega}{2\pi}, \quad I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

Variables

L	Span between bearings	m
d	Shaft diameter	m
E	Young's modulus	Pa
ρ	Shaft density	kg/m³
m	Mass of a disc at mid-span	kg
N_s	Critical speed of the shaft alone	1/s
N_d	Critical speed of the disc on a massless shaft	1/s
N_c	Combined critical speed (Dunkerley)	1/s
I	Second moment of area of the shaft	m⁴
A	Cross-section area of the shaft	m²
$\omega_s, \omega_d, \omega_c$	Angular critical speeds (shaft, disc, combined)	rad/s

Method and standards

Beam vibration and Dunkerley's rule.

Assumptions and limits

- Simply supported, uniform, solid round shaft of any material (E and ρ are inputs); hollow shafts are not covered.
- The disc is a point mass at mid-span; its gyroscopic effect and the flexibility of bearings and supports are ignored (both change the real critical speed, flexible supports lower it).
- With no disc, N_d is not applicable and is not shown.

References

• Euler–Bernoulli beam: ω = (π/L)² √(EI/ρA) for a simply supported shaft; Dunkerley (1894)

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/instrumentation/vibration-noise/shaft-critical-speed

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.