Eigenvalues of a 3×3 Matrix

Eigenvalues of a real 3×3 matrix from its characteristic cubic, including a complex pair, with the trace, determinant and second invariant.

Method: Characteristic polynomial and cubic solution

Input parameters

Unit system: SI

Parameter	Value	Unit
Element row 1, column 1	4.0000	
Element row 1, column 2	1.0000	
Element row 1, column 3	2.0000	
Element row 2, column 1	1.0000	
Element row 2, column 2	3.0000	
Element row 2, column 3	0	
Element row 3, column 1	2.0000	
Element row 3, column 2	0	
Element row 3, column 3	5.0000	

Calculated results

Parameter	Value	Unit
Eigenvalue 1 (largest real)	6.6691	
Eigenvalue 2 (real part)	3.4760	
Eigenvalue 3 (real part)	1.8549	
Imaginary part of a complex pair (± ; zero if all real)	0	
Trace	12.000	
Second invariant	42.000	
Determinant	43.000	

Notes for this case

• The eigenvalues are the roots of $\det(A - \lambda I) = \lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$ with the invariants trace I_1, sum of principal 2×2 minors I_2 and determinant I_3. The cubic is reduced to $t^3 + pt + q = 0$ and solved by the trigonometric method when all roots are real (always so for a symmetric matrix such as a stress or inertia tensor) or by Cardano's formula when there is one real root and a complex pair $\lambda_{2,3}$ = Re ± i·Im. Repeated roots are sensitive to rounding in the inputs.

Formula and method

$$\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$$

Variables

a_{11}	Element row 1, column 1	-
a_{12}	Element row 1, column 2	-
a_{13}	Element row 1, column 3	-
a_{21}	Element row 2, column 1	-
a_{22}	Element row 2, column 2	-
a_{23}	Element row 2, column 3	-
a_{31}	Element row 3, column 1	-
a_{32}	Element row 3, column 2	-

a_{33}	Element row 3, column 3	-
λ_1	Eigenvalue 1 (largest real)	-
λ_2	Eigenvalue 2 (real part)	-
λ_3	Eigenvalue 3 (real part)	-
$\mathrm{Im}\,\lambda_{2,3}$	Imaginary part of a complex pair (± ; zero if all real)	-
I_1	Trace	-
I_2	Second invariant	-
I_3	Determinant	-

Method and standards

Eigenvalues.

Assumptions and limits

- Real 3×3 matrix; double precision.

References

Characteristic polynomial and cubic solution

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/general/geometry/eigenvalues-3x3-matrix

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Reopens the calculator with the inputs of this report.