Cubic Equation Roots

Real and complex roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ by the trigonometric and Cardano methods, with the discriminant and the number of real roots.

Method: Cardano and trigonometric solutions of the cubic

Input parameters

Unit system: SI

Parameter	Value	Unit
Coefficient a (x^3)	1.0000	
Coefficient b (x^2)	-6.0000	
Coefficient c (x)	11.000	
Constant d	-6.0000	

Calculated results

Parameter	Value	Unit
Root 1 (always real)	3.0000	
Root 2 (real part if complex)	2.0000	
Root 3 (real part if complex)	1.00000	
Imaginary part of the complex pair (0 if all real)	0	
Discriminant	4.0000	
Number of distinct real roots	3.0000	

Notes for this case

• Dividing by a and substituting x = t − b/3a gives the depressed cubic $t^3 + pt + q = 0$. With three real roots (discriminant positive) the trigonometric form t = 2√(−p/3)·cos[(1/3)acos(3q/(2p)·√(−3/p)) − 2πk/3] avoids complex arithmetic; with one real root Cardano's formula t = ∛(−q/2 + √Δ′) + ∛(−q/2 − √Δ′) gives it and the complex pair. Check roots by substitution; equal roots are sensitive to rounding.

Formula and method

$$x = t - \frac{b}{3a}, \quad t^3 + pt + q = 0, \quad t_k = 2\sqrt{-\frac{p}{3}}\cos\left[\frac{1}{3}\cos^{-1}\left(\frac{3q}{2p}\sqrt{-\frac{3}{p}}\right) - \frac{2\pi k}{3}\right]$$

Variables

a	Coefficient a (x^3)	-
b	Coefficient b (x^2)	-
c	Coefficient c (x)	-
d	Constant d	-
x_1	Root 1 (always real)	-
x_2	Root 2 (real part if complex)	-
x_3	Root 3 (real part if complex)	-
$\pm\Im$	Imaginary part of the complex pair (0 if all real)	-
Δ	Discriminant	-
n_{real}	Number of distinct real roots	-

Method and standards

Cubic roots.

Assumptions and limits

- Real coefficients.

References

Cardano and trigonometric solutions of the cubic

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/general/geometry/cubic-equation-roots

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Reopens the calculator with the inputs of this report.