Arithmetic Gradient Series: Present Worth and Annuity

Present worth and equivalent annual amount of cash flows that start at a base amount and grow by a constant amount each year, at an interest rate.

Method: Arithmetic gradient factors

Input parameters

Unit system: SI

Parameter	Value	Unit
Base amount in year 1	50,000	
Increase per year (gradient)	4,000.0	
Interest (discount) rate	8.0000	%
Number of years	10.000	

Calculated results

Parameter	Value	Unit
Present worth of the base series	335,504	
Present worth of the gradient	103,907	
Total present worth	439,411	
Equivalent annual of the gradient	15,485	
Total equivalent annual amount	65,485	

Notes for this case

• Year t's cash flow is A_1 + (t − 1)G. The uniform part uses (P/A, i, n) = [(1 + i)^n − 1]/[i(1 + i)^n]; the gradient part (P/G, i, n) = (1/i)[(P/A) − n/(1 + i)^n], and its equivalent annual (A/G) = 1/i − n/[(1 + i)^n − 1]. Typical uses: rising maintenance costs, growing savings or declining output (negative G).

Formula and method

$$\left(\frac{P}{G}\right) = \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right], \quad \left(\frac{A}{G}\right) = \frac{1}{i} - \frac{n}{(1+i)^n - 1}$$

Variables

A_1	Base amount in year 1	-
G	Increase per year (gradient)	-
i	Interest (discount) rate	-
n	Number of years	-
P_A	Present worth of the base series	-
P_G	Present worth of the gradient	-
P	Total present worth	-
A_G	Equivalent annual of the gradient	-
A	Total equivalent annual amount	-

Method and standards

Gradient series.

Assumptions and limits

- End-of-year cash flows; constant interest rate.

References

Arithmetic gradient factors

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/general/economics/arithmetic-gradient-series

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Reopens the calculator with the inputs of this report.