Granular Filter Particle Removal: Yao and Tufenkji–Elimelech Models

Single-collector contact efficiency by interception, sedimentation and diffusion (Yao and Tufenkji–Elimelech models) and the clean-bed particle removal.

Input parameters

Unit system: SI

Parameter	Value	Unit
Particle diameter	1.0000	µm
Filter grain (collector) diameter	0.60000	mm
Filtration rate	10.000	m/h
Bed depth	0.75000	m
Bed porosity	42.000	%
Attachment efficiency	100.00	%
Particle density	1,050.0	kg/m³
Water density	999.10	kg/m³
Water viscosity	1.1380	mPa·s
Water temperature	15.000	°C
Hamaker constant, 10^{-20} J	1.0000	

Calculated results

Parameter	Value	Unit
Relative size group d_p/d_c	0.001667	
Gravity number	8.7726e-06	
Peclet number	4,493,246	
Attraction number	3.3565e-04	
van der Waals number	2.5136	
Happel porosity function	33.639	
TE interception efficiency	1.5120e-04	
TE sedimentation efficiency	2.6138e-06	
TE diffusion efficiency	2.3901e-04	
Total single-collector efficiency, Tufenkji–Elimelech	3.9282e-04	
Total single-collector efficiency, Yao	1.5984e-04	
Clean-bed passing fraction, TE model	65.234	%
Log removal, TE model	0.18553	
Clean-bed passing fraction, Yao model	84.044	%

Notes for this case

- Clean-bed removal $\ln(C/C_0) = -3(1 - \varepsilon)\alpha\eta L/(2d_c)$ for monodisperse media; the attachment efficiency α depends on coagulation (near 1 when particles are well destabilised).

Formula and method

$$\eta_{TE} = 0.55 A_S N_A^{1/8} N_R^{1.675} + 0.22 N_R^{-0.24} N_{vdW}^{0.053} N_G^{1.11} + 2.4 A_S^{1/3} N_R^{-0.081} N_{vdW}^{0.052} Pe^{-0.715}$$

$$\eta_{Yao} = \frac{3}{2} N_R^2 + N_G + 4 Pe^{-2/3}, \quad \frac{C}{C_0} = \exp\left[-\frac{3(1-\varepsilon)\alpha\eta L}{2 d_c}\right]$$

Variables

d_p	Particle diameter	m
d_c	Filter grain (collector) diameter	m
v_F	Filtration rate	m/s
L	Bed depth	m
ε	Bed porosity	-
α	Attachment efficiency	-
ρ_p	Particle density	kg/m³
ρ_w	Water density	kg/m³
μ	Water viscosity	Pa·s
T	Water temperature	K
H_a	Hamaker constant, 10^{-20} J	-
N_R	Relative size group d_p/d_c	-
N_G	Gravity number	-
Pe	Peclet number	-
N_A	Attraction number	-
N_{vdW}	van der Waals number	-
A_S	Happel porosity function	-
η_I	TE interception efficiency	-
η_G	TE sedimentation efficiency	-
η_D	TE diffusion efficiency	-
η_{TE}	Total single-collector efficiency, Tufenkji–Elimelech	-
η_{Yao}	Total single-collector efficiency, Yao	-
C/C_0	Clean-bed passing fraction, TE model	-
LRV	Log removal, TE model	-
$C/C_0\,(Yao)$	Clean-bed passing fraction, Yao model	-

Method and standards

Colloid filtration theory with the Yao and Tufenkji–Elimelech correlations for single-collector efficiency.

Assumptions and limits

- Clean bed, monodisperse media, favourable attachment.

Report details

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This calculator online

https://calc.firecad.com/environmental/drinking-water/filter-single-collector-efficiency

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Reopens the calculator with the inputs of this report.