Ellipsoid Radii of Curvature at a Latitude

Meridian and prime vertical radii of curvature, Gaussian mean radius and the radius in a given azimuth on the GRS80 (or another) ellipsoid at a latitude.

Method: Ellipsoid geometry

Input parameters

Unit system: SI

Parameter	Value	Unit
Semi-major axis	6,378.1	km
Inverse flattening	298.26	
Latitude	40.000	°
Azimuth of the line	45.000	°

Calculated results

Parameter	Value	Unit
Meridian radius of curvature	6,361.8	km
Prime vertical radius of curvature	6,387.0	km
Gaussian mean radius √(MN)	6,374.4	km
Radius in the azimuth	6,374.4	km
Length of 1″ of latitude	30.843	m
Length of 1″ of longitude	23.721	m

Notes for this case

• e² = f(2 − f); M = a(1 − e²)/(1 − e²sin²φ)^{3/2} along the meridian, N = a/(1 − e²sin²φ)^{1/2} perpendicular to it (Euler: 1/R_α = cos²α/M + sin²α/N). The Gaussian mean √(MN) is used to reduce distances to the ellipsoid and in map projection scale factors. Defaults are GRS80 (NAD 83, ETRS89, ITRF practice); WGS 84 differs only slightly (1/f = 298.257223563).

Formula and method

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}, \quad N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}, \quad \frac{1}{R_\alpha} = \frac{\cos^2\alpha}{M} + \frac{\sin^2\alpha}{N}$$

Variables

a	Semi-major axis	m
$1/f$	Inverse flattening	-
φ	Latitude	rad
α	Azimuth of the line	rad
M	Meridian radius of curvature	m
N	Prime vertical radius of curvature	m
R_m	Gaussian mean radius √(MN)	m
R_α	Radius in the azimuth	m
$\Delta s_{1''\varphi}$	Length of 1″ of latitude	m
$\Delta s_{1''\lambda}$	Length of 1″ of longitude	m

Method and standards

Radii of curvature.

Assumptions and limits

- Ellipsoid of revolution.

References

Ellipsoid geometry

Report details

Report reference	FCC-XST9-PRRX (the same inputs always give the same reference)
Generated	23 November 2026, 18:09 UTC
Calculator	Ellipsoid Radii of Curvature at a Latitude
Method last reviewed	17 November 2026
Software release	FireCAD Calc, release of 22 November 2026
Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/civil/surveying/ellipsoid-radii-of-curvature

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.