River Oxygen Sag by Streeter–Phelps

Critical time, distance and dissolved oxygen deficit below a wastewater discharge into a river, from BOD, initial deficit and deoxygenation and reaeration rates.

Input parameters

Unit system: SI

Parameter	Value	Unit
Ultimate BOD of the mixed river water	20.000	mg/L
Initial oxygen deficit	1.0000	mg/L
Deoxygenation rate (base e)	0.23000	1/d
Reaeration rate (base e)	0.50000	1/d
Saturation dissolved oxygen	9.1000	mg/L
River velocity	0.30000	m/s

Calculated results

Parameter	Value	Unit
Critical time	2.6520	d
Critical distance	68.740	km
Critical (maximum) deficit	4.9990	mg/L
Minimum dissolved oxygen	4.1010	mg/L
Self-purification factor k_2/k_1	2.1739	

Notes for this case

• Downstream of a discharge, BOD exertion (k_1L) uses oxygen while reaeration (k_2D) restores it; the deficit rises to a maximum at the critical time and then recovers. The critical point is where the two rates balance, D_c = (k_1/k_2)L_0e^(−k_1t_c). Rates must be at the river temperature (k_1 about 0.1–0.4 per day; k_2 from 0.1 for sluggish rivers to over 1 for rapids, e.g. by O'Connor–Dobbins 3.9u^0.5/H^1.5). Compare the minimum DO with the standard for the river's use.

Formula and method

$$t_c = \frac{1}{k_2 - k_1} \ln\left[\frac{k_2}{k_1}\left(1 - \frac{D_0(k_2 - k_1)}{k_1 L_0}\right)\right], \quad D_c = \frac{k_1}{k_2} L_0 e^{-k_1 t_c}$$

Variables

L_0	Ultimate BOD of the mixed river water	kg/m³
D_0	Initial oxygen deficit	kg/m³
k_1	Deoxygenation rate (base e)	1/s
k_2	Reaeration rate (base e)	1/s
C_s	Saturation dissolved oxygen	kg/m³
u	River velocity	m/s
t_c	Critical time	s
x_c	Critical distance	m
D_c	Critical (maximum) deficit	kg/m³
DO_{min}	Minimum dissolved oxygen	kg/m³
f	Self-purification factor k_2/k_1	-

Method and standards

Oxygen sag.

Assumptions and limits

- Single point discharge; steady uniform flow.

Report details

Report reference	FCC-43VN-P11T (the same inputs always give the same reference)
Generated	23 November 2026, 17:51 UTC
Calculator	River Oxygen Sag by Streeter–Phelps
Method last reviewed	17 November 2026
Software release	FireCAD Calc, release of 22 November 2026
Release checks	3 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/civil/hydraulics/streeter-phelps-oxygen-sag

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.