Underwood Minimum Reflux, Multicomponent (Up to 4)

Minimum reflux of a multicomponent distillation by the Underwood equations: the common root θ between the key volatilities, then Rmin from the distillate.

Method: Perry's Chemical Engineers' Handbook; 9th ed. (McGraw-Hill, 2019); ...

Input parameters

Unit system: SI

Parameter	Value	Unit
Feed condition q	1.0000	
Light key is component	2	
α of component 1 (relative to any one reference component)	4.0000	
α of component 2	2.0000	
α of component 3	1.0000	
α of component 4 (0 = absent)	0	
Feed mole fraction of 1	20.000	%
Feed mole fraction of 2	40.000	%
Feed mole fraction of 3	40.000	%
Feed mole fraction of 4	0	%
Distillate mole fraction of 1	33.000	%
Distillate mole fraction of 2	65.000	%
Distillate mole fraction of 3	2.0000	%
Distillate mole fraction of 4	0	%

Calculated results

Parameter	Value	Unit
Underwood root	1.2835	
Minimum reflux ratio	1.2297	

Notes for this case

- The distillate composition should be the one at minimum reflux; with sharp key splits the non-keys lie wholly in one product.

Formula and method

$$\sum_i \frac{\alpha_i z_i}{\alpha_i - \theta} = 1 - q, \quad R_{min} + 1 = \sum_i \frac{\alpha_i x_{D,i}}{\alpha_i - \theta}$$

Variables

q	Feed condition q	-
LK	Light key is component	-
α_1	α of component 1 (relative to any one reference component)	-
α_2	α of component 2	-
α_3	α of component 3	-
α_4	α of component 4 (0 = absent)	-
z_1	Feed mole fraction of 1	-
z_2	Feed mole fraction of 2	-
z_3	Feed mole fraction of 3	-

z_4	Feed mole fraction of 4	-
$x_{D,1}$	Distillate mole fraction of 1	-
$x_{D,2}$	Distillate mole fraction of 2	-
$x_{D,3}$	Distillate mole fraction of 3	-
$x_{D,4}$	Distillate mole fraction of 4	-
θ	Underwood root	-
R_{min}	Minimum reflux ratio	-

Method and standards

Underwood equations solved by bisection for the root between the keys.

Assumptions and limits

- Constant α and molar overflow.
- Volatilities may be relative to any one component (a value below 1 is allowed for components heavier than the reference), in falling order.
- Feed and distillate fractions within ±1 % of 1 are accepted as entered, without normalising.
- Both keys must be in the feed, and a root must be found between their volatilities.

References

• Perry's Chemical Engineers' Handbook
• 9th ed. (McGraw-Hill, 2019)
• sec. 13 (distillation)
• Underwood equations (13-37) and (13-38)

Report details

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Release checks	2 reference cases of this calculator, computed independently, are checked on every release

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

https://calc.firecad.com/chemical/separation/underwood-multicomponent-minimum-reflux

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.