# Hill Equation: Cooperativity from Two Points

Hill coefficient and half-saturation constant from two measured fractional saturations (or responses), and the saturation at another concentration.

**Method:** Hill equation and Hill plot

## Input parameters

Unit system: SI

| Parameter | Value | Unit |
|---|---|---|
| First concentration | 0.020000 | mmol/L |
| Fractional saturation at the first | 20.000 | % |
| Second concentration | 0.050000 | mmol/L |
| Fractional saturation at the second | 70.000 | % |
| Concentration for the prediction | 0.040000 | mmol/L |

## Calculated results

| Parameter | Value | Unit |
|---|---|---|
| Hill coefficient | 2.4376 | |
| Concentration at half saturation | 0.035319 | mmol/L |
| Saturation at the concentration for the prediction | 57.526 | % |

### Notes for this case

• The Hill equation θ = $S^n/(K_{0.5}^n + S^n)$ describes binding or response with cooperativity: n > 1 means binding of one ligand helps the next (a sigmoidal curve, as oxygen on haemoglobin with n ≈ 2.8); n = 1 is ordinary Michaelis–Menten behaviour. Taking logs, $\ln[\theta/(1-\theta)] = n \ln S - n \ln K_{0.5}$, a straight line through the two points.

## Formula and method

$$\theta = \frac{S^n}{K_{0.5}^n + S^n}, \quad n = \frac{\ln \frac{\theta_1/(1-\theta_1)}{\theta_2/(1-\theta_2)}}{\ln(S_1/S_2)}$$

### Variables

| | | |
|---|---|---|
| $S_1$ | First concentration | mol/m³ |
| $\theta_1$ | Fractional saturation at the first | - |
| $S_2$ | Second concentration | mol/m³ |
| $\theta_2$ | Fractional saturation at the second | - |
| $S$ | Concentration for the prediction | mol/m³ |
| $n_H$ | Hill coefficient | - |
| $K_{0.5}$ | Concentration at half saturation | mol/m³ |
| $\theta$ | Saturation at the concentration for the prediction | - |

### Method and standards

Hill equation.

### Assumptions and limits

- Two-parameter Hill model.

### References

Hill equation and Hill plot

## Report details

| | |
|---|---|
| Report reference | FCC-DQWR-9NV2 (the same inputs always give the same reference) |
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| Calculator | Hill Equation: Cooperativity from Two Points |
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| Release checks | 3 reference cases of this calculator, computed independently, are checked on every release |

Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

### This calculator online

https://calc.firecad.com/chemical/bioprocess/hill-equation-fit

Open this case on calc.firecad.com →

Reopens the calculator with the inputs of this report.