Thick Cylinder: Thermal and Pressure Hoop Stress

Inner and outer hoop stresses of a thick cylinder (drum, header) from a steady through-wall temperature difference plus internal pressure (Lamé).

Method: Thermal stresses in a long thick cylinder with a steady logarithmic temperature profile (Timoshenko and Goodier, Theory of Elasticity, 3rd ed., McGraw-Hill, 1970); Lamé pressure stresses

Input parameters

Unit system: SI

Parameter	Value	Unit
Inner radius	0.60000	m
Outer radius	0.69000	m
Inner minus outer surface temperature	30.000	K
Internal pressure (gauge)	45.000	bar
Young's modulus	195.00	GPa
Linear expansion coefficient	13.000	µm/(m·K)
Poisson's ratio	0.30000	

Calculated results

Parameter	Value	Unit
Thermal hoop stress, inner surface	-56.849	MPa
Thermal hoop stress, outer surface	51.794	MPa
Pressure hoop stress, inner surface	32.407	MPa
Pressure hoop stress, outer surface	27.907	MPa
Total hoop stress, inner surface	-24.442	MPa
Total hoop stress, outer surface	79.701	MPa

Notes for this case

- Steady logarithmic temperature profile; during start-up the transient profile differs (see the drum warm-up calculator). Design limits come from the applicable code.

Formula and method

$$\sigma_{\theta,T}(r_i) = \frac{E\beta\Delta T}{2(1-\nu)\ln(r_o/r_i)}\left[1 - \frac{2r_o^2}{r_o^2 - r_i^2}\ln\frac{r_o}{r_i}\right], \quad \sigma_{\theta,p}(r_i) = p\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}$$

$$\sigma_{\theta,T}(r_o) = \frac{E\beta\Delta T}{2(1-\nu)\ln(r_o/r_i)}\left[1 - \frac{2r_i^2}{r_o^2 - r_i^2}\ln\frac{r_o}{r_i}\right], \quad \sigma_{\theta,p}(r_o) = p\frac{2r_i^2}{r_o^2 - r_i^2}, \quad \sigma_\theta = \sigma_{\theta,T} + \sigma_{\theta,p}$$

Variables

r_i	Inner radius	m
r_o	Outer radius	m
ΔT	Inner minus outer surface temperature	K
p	Internal pressure (gauge)	Pa
E	Young's modulus	Pa
β	Linear expansion coefficient	1/K
ν	Poisson's ratio	-
$\sigma_{\theta,T}(r_i)$	Thermal hoop stress, inner surface	Pa

$\sigma_{\theta,T}(r_o)$	Thermal hoop stress, outer surface	Pa
$\sigma_{\theta,p}(r_i)$	Pressure hoop stress, inner surface	Pa
$\sigma_{\theta,p}(r_o)$	Pressure hoop stress, outer surface	Pa
$\sigma_{\theta}(r_i)$	Total hoop stress, inner surface	Pa
$\sigma_{\theta}(r_o)$	Total hoop stress, outer surface	Pa

Method and standards

Thermal hoop stresses of a long cylinder with a steady logarithmic temperature distribution through the wall (Timoshenko and Goodier), added to the Lamé hoop stresses from internal gauge pressure at the inner and outer surfaces.

Assumptions and limits

- Long cylinder, elastic, steady.
- Hoop stresses only. The axial thermal stress (equal to the hoop thermal stress at the surfaces for a long cylinder with free ends) and the radial pressure stress (−p inside) are not shown, so no equivalent (Tresca or von Mises) stress is given for comparison with allowables.

References

• Thermal stresses in a long thick cylinder with a steady logarithmic temperature profile (Timoshenko and Goodier, Theory of Elasticity, 3rd ed., McGraw-Hill, 1970); Lamé pressure stresses

Report details

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Results are for engineering estimation. Verify critical designs against the referenced standards and your own data.

This calculator online

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